

Multivariate Regression Analysis: Considering the Relevance of Past Performance

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Abstract

Investors often use past performance as a major source of knowledge about an asset class or a particular investment manager. Past performance can tell us a lot about the tendencies of asset classes and managers, but its meaning should be evaluated with great care. Simply comparing performances over an arbitrary time period can give way to a pattern of return chasing that can severely detract from performance. In fact, the emotional behavior of changing investments based on past returns has been the topic of many publications and hours of research. This paper will provide a statistical perspective on the relevance of past performance. Specifically, I will show that multivariate regression analysis can successfully identify mathematical relationships between various past performance statistics and future returns.

Multivariate regression analysis is not a foreign concept to the financial industry. It has been utilized by technical and quantitative analysts for some time now. However, constructing and interpreting this type of statistical analysis can be obstacles to investors without technical backgrounds. With this in mind, I will give a description of each of the variables and results in the analysis and inform the reader of what is necessary for a statistically significant prediction model. I believe that this insight will make the benefits of multivariate regression analysis accessible to a wider variety of investors.

I begin with a concise description of the assets used as representation for

the sectors of the S&P 500. This description is followed by an explanation of the method used for calculating returns and the frequency with which they are calculated. These two variables, calculation and frequency, are often debated topics and can have material effects on the outcome of the analyses. In this paper, I will use a monthly return frequency to calculate a logarithmic return. These decisions are key to multivariate regression analysis and must be made before further analysis is completed, making comparison between the outcomes of using different frequencies and return calculations quite time-consuming. However, it does force this decision to be a forethought and lessens the bias that could be present if it were an afterthought.

Investors often select calculation time periods with hindsight bias by comparing different returns and selecting the one that looks the best at that point in time. To better answer the questions, “What time period should be used for performance calculations?” And, “How long is the prediction good for?” I will attach statistical significance to four different look back time periods and four different future time periods and make an informed decision on which combination has the highest predictive capability. By modeling each of these different combinations of time periods, we gain insight about the sensitivity of the analysis to the time period variable. I will show that varying levels of significance exists across the different time period combinations and select one pair to be the optimal time period combination. I will discuss results from each look back analysis, but for

brevity this will not be an exhaustive exposition.

Finally, I will display the predictive capability of employing a multivariate regression model from the optimal time period combination in an actively managed sector rotation trading system. The performance that results from this trading system outperforms the S&P 500 on a risk adjusted basis according to several well-accepted performance measures. This success is mainly due to the down side protection incurred by rotating through the US equity sectors via a rules-based decision making process, while still participating on the up side. For further analytical rigor, I forward tested the trading strategy 36 months to verify the analysis with out-of-sample data. I will show that the trends discovered by multivariate regression analysis are also present in data excluded from the backtest. Ultimately, I will show that a rules-based trading system, supplemented with multivariate regression, is a viable alternative to investing in a passive index.

1 Introduction

Today, there are approximately \$7.8 trillion in assets benchmarked to the S&P 500 with \$2.2 trillion invested in funds that seek to replicate the return of the Index [SPIndices]. I believe that too many investors have given up on an actively managed large cap allocation in their portfolios in favor of a market-cap weighted index fund. By electing to invest in the S&P 500 Index, whether through a mutual fund or ETF, investors have chosen to merely participate in the stock market while there is ample opportunity to outperform.

The allocation of the S&P 500 index is devised among 500 US large cap companies, ranked by market cap. Approximately 80% of the entire US market is contained within this index [SPIndices]. The S&P 500 is rebalanced occasionally and is monitored by a committee in accordance to the methodology laid out by S&P Dow Jones Indices [SPIndices]. As of December, 2015, the market-cap allocation (by GICS sector) is found in Table 1.

As you can see, over 50% of the S&P 500 is allocated to three sectors: Information Technology, Financials, and Healthcare. Due to their large weighting, the

Table 1: Current S&P 500 GICS Sector Allocations

Information Technology	20.9%
Financials	16.6%
Healthcare	14.6%
Consumer Discretionary	13.1%
Industrials	10.1%
Consumer Staples	9.6%
Energy	7.1%
Utilities	2.9%
Telecommunications	2.3%

performance of these sectors has a greater impact on the return of the S&P 500 than those with smaller weighting. The Utilities sector, for example, had the highest monthly return 17 times in the last 5 years; Healthcare, Technology, and Financials, had a *combined* total of 17 highest monthly returns over the same time period.¹ During the 17 months in which Utilities had the highest return, its excess return to the S&P 500 Index, on average, was over 4% per month! Since Utilities makes up less than 3% of the Index, its performance is hardly realized in an S&P 500 Index fund. This consequence is the basis for a sector rotation strategy and presents a question only an active manager can address, “How do I allocate to the right sector(s) at the right time?”

2 Underlying Data

2.1 Select Sector SPDR ETFs

I have chosen to use the total returns from the nine Select Sector SPDR ETFs as representation of the sectors that make up the S&P 500 Index. The returns were taken from Morningstar Direct² and span a total of nearly 17 years (1/1/1999 to 11/30/2015). In order to maintain a uniform ending point for each analysis, the calculations end on 11/30/2012 (three years prior to the end of our data set).

A list of the Select Sector SPDR ETFs is found in Table 2. Note that the Select Sector ETFs do not exactly replicate the GICS sectors found in Table 1, but for the purpose of this paper the Select Sector SPDR ETFs achieve the same effect,

¹Monthly return rankings calculated from monthly return data provided by Morningstar from December '10 to November '15.

²Morningstar Direct is a cloud-based investment analysis platform that provides access to institutional quality data, analytics, and research.

Table 2: SPDR Select Sectors

XLV	Healthcare
XLI	Industrials
XLV	Consumer Discretionary
XLP	Consumer Staples
XLB	Materials
XLK	Technology
XLU	Utilities
XLE	Energy
XLFX	Financials

which is to divide the S&P 500 holdings into non-overlapping subsets. As a matter of fact, the Select Sector SPDR ETFs were developed exactly for this purpose: to allow investors to construct their own allocation of the well-know large cap S&P 500 stocks based on their specific investment goals and strategies [SectorSPDR].

2.2 Return Calculation

The two most common measures of return are the geometric average return and the arithmetic average return. However, most discussion between the two may not adequately advise practitioners about the proper use of these concepts when forecasting future returns [Hughson et. al., 2006].

The input data in this paper includes a future cumulative return variable and therefore requires sensitivity with regard to the calculation used. University of Colorado, Boulder professors Eric Hughson, Chris Yung, and Michael Stutzer made a statement on the topic of forecasting returns:

Those wanting to forecast a typical future cumulative return should be more interested in estimating the median future cumulative return than in estimating the mathematical expected cumulative return. For that pur-

pose, continuous compounding of the mathematical expected logarithmic return is more relevant than ordinary compounding of the mathematical expected return [Hughson et. al., 2006].

Therefore, I have chosen to use the logarithmic return calculation. The logarithmic return R is defined as

$$R = \ln\left(\frac{V_f}{V_i}\right) \quad (1)$$

where V_i is the value of the asset at the beginning time period, V_f is the value of the asset at the ending time period, and $\ln()$ is the natural log function.

A discussion on the frequency used for measuring the return will not be presented in this paper. For practical purposes, I have chosen to use a monthly return interval as the foundation for the analyses.

3 Introducing Multivariate Regression

Here, I will describe some of the key terms involved in multivariate regression. Stata's³ **mvreg** (multivariate regression) command takes the independent and dependent variables for each sector at every time period and finds a straight line that best fits the data. Then, as output, **mvreg** calculates a regression coefficient⁴ for each independent variable and gives a summary of how well the overall model and each variable predicted the future returns.

³Stata is a general-purpose statistical software package used for data management, statistical analysis, graphics, simulations, regression, and custom programming.

⁴A coefficient is a multiplicative number in an equation. For example, in $2x - 0.5y$ the coefficients of x and y are 2 and 0.5, respectively.

3.1 Independent Variables

I chose these five independent variables for their logical application to how investors evaluate assets.

Return was selected as an independent variable because investors often make choices based on the historical returns of an asset. These multivariate regression analyses will help put some science behind the claim, “Past performance cannot guarantee future results.” Each trailing return variable will be defined using the same calculation as Equation (1)

$$Return_{t-x}(Sector) = \ln\left(\frac{V_t}{V_{t-x}}\right)$$

where x is the look back period and t is the current month.

Volatility is often associated with risk and investors often measure the performance of an asset based on how well they are compensated for that risk. I used sample standard deviation instead of the population standard deviation due to the small number of observations used in each calculation, this is defined as

$$s_{t-x}(Sector) = \sqrt{\frac{1}{N-1} \sum_{i=t-x}^t (x_i - \bar{x})^2}$$

where x is the look back period, t is the current month, $x_i = x_{t-x}, \dots, x_t$ are the observed monthly logarithmic returns, \bar{x} is the mean of these values, and N is the size of the sample. In Microsoft Excel, this is simply computed by the function “STDEV.S()”.

Drawdown, which is also used as a measure of risk, is defined as

$$Drawdown_{t-x}(Sector) = \text{Min}(0, Return_{t-x}(Sector))$$

where $\text{Min}(y, z)$ is a function that chooses the smallest of the two variables, x is the look back period, and t is the current month. Essentially, $Drawdown_{t-x}$ will be set equal to 0 if $Return_{t-x}$ is positive or it will be set equal to $Return_{t-x}$ if $Return_{t-x}$ is negative. This will give us an idea as to whether downturns in the market are likely to continue, or if there is some form of mean reversion⁵ taking place.

The regression slope is a function defined as

$$LinRegSlope_{t-x}(Sector) = \text{LINEST}(V_{t-x}, \dots, V_t)$$

where x is the look back period and t is the current month. “LINEST()” is a function in Microsoft Excel that calculates the statistics for a line by using the “least squares” method to find the slope of a best fit straight line through the values . This measure was chosen as a way to quantify the rate of change over time in the price of an asset.

The fifth and final independent variable is the excess return between the respective sector and the S&P 500, which is simply

$$EXRET_{t-x}(Sector) = Return_{t-x}(Sector) - Return(S\&P500)_{t-x}$$

where x is the look back period and t is the current month. If the past return of the asset is greater than the S&P 500, this will be a positive number, and if it is less

⁵Mean reversion is the assumption that an asset’s price will tend to move to the average price over time.

than the S&P 500, it will be negative.

3.2 Dependent Variables

This analysis attempts to explain the variation in the dependent variable using the independent variables at each new time period. Specifically, the dependent variables are the future returns of each sector over four different time periods: 3-months, 6-months, 12-months, and 36-months. They are calculated using the equation

$$Return_{t+k}(Sector) = \ln\left(\frac{V_{t+k}}{V_t}\right)$$

where V_t and V_{t+k} are the asset values at the beginning and the end of the future return length, respectively.

3.3 Regression Coefficients and Formula

Now, I will define each of the regression coefficients as well as present the regression formula using an x -month look back and a k -month future return.

- Define β_0 as the intercept⁶,
- Define β_1 as the coefficient for $Return_{t-x}$,
- Define β_2 as the coefficient for s_{t-x} ,
- Define β_3 as the coefficient for $Drawdown_{t-x}$,
- Define β_4 as the coefficient for $LinRegSlope_{t-x}$,

⁶The intercept, or constant, value of a regression model is the mean of the dependent variable when all independent variables are set to 0.

- Define β_5 as the coefficient for $EXRET_{t-x}$, and
- Define $Return_{t+k}$ as the prediction of return over the next k months.

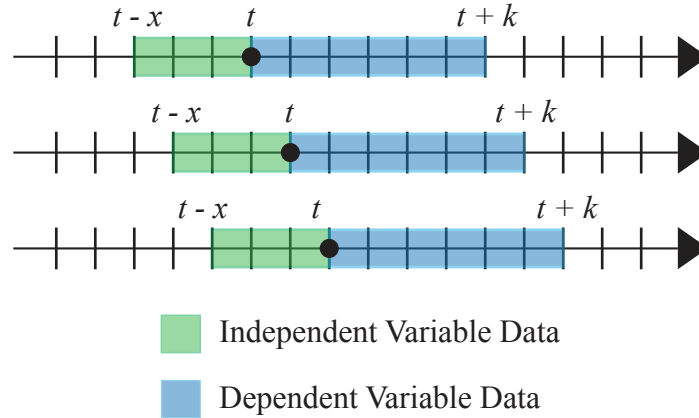
The regression coefficients and independent variables are plugged into the regression equation for $Return_{t+k}$:

$$Return_{t+k} = \beta_0 + \beta_1 Return_{t-x} + \beta_2 s_{t-x} + \beta_3 Drawdown_{t-x} + \beta_4 LinRegSlope_{t-x} + \beta_5 EXRET_{t-x}.$$

This is the standard multivariate regression model formula. The right hand side of this equation includes the regression coefficients suggested by **mvreg** and are each multiplied by their respective independent variable. The left hand side of this equation (the result) is the predicted value for the specified future return.

Figure 1 gives a visualization of the calculation process. One month at a time, the multivariate regression analysis uses the independent variables to attempt to find a mathematical relationship to the dependent variables. Each of these statistical models are summarized by various measures of fit.

Figure 1: As the current time period changes, so do the time periods that are used to calculate the input variables.



3.4 Measuring Goodness of Fit

To answer the question, “**Which** predictions are valid and **why**?” I will explain several measures of “goodness of fit.”

The root mean squared error (RMSE), the coefficient of determination (R^2), and the F-ratio are all calculated by **mvreg** to suggest which of the overall models have predictive capability.

The RMSE, also called forecasting error, is the spread between the actual future returns and the predicted future returns. In other words, it is the average distance between the best fit line and the predicted returns. RMSE is always between 0 and 1 and its significance increases as it gets closer to 0. RMSE is only used to compare the forecasting errors of different time periods for a particular Select Sector SPDR ETF and not between the Select Sector SPDR ETFs.

R^2 is a popular measure in portfolio management and a key output of regression analysis. This value explains the proportion of the variance in the future return that is predictable from the independent variables. The R^2 value presented in this paper is the Adjusted R^2 value⁷. This measure is always between 0 and 1 and its significance increases as the value gets closer to 1.

The F-ratio is used to decide whether the overall model has statistically significant predictive capability. To determine the significance of the F-ratio, we can look at its associated p-value⁸. In this paper, we will look for p-values smaller than 0.01. If the F-ratio calculated by **mvreg** is significant, we can infer that the overall model has

⁷If R^2 is not adjusted, it increases with each additional independent variable included in the calculation which can mislead results.

⁸For a more detailed description of how the F-ratio and associated p-value are calculated, please refer to [Winter, 2015]

predictive capability.

After deciding **which** overall models have attractive measures of goodness of fit, we “drill” down into the model to look at the regression coefficient and t-statistic of each independent variable to determine **why**.

The regression coefficient is a factor that determines how each independent variable affects the dependent variable. For example, if the past return has a negative regression coefficient, the past return is modeled to **decrease** the prediction of the future return (mean reversion). In the opposite case, if the past return has a positive regression coefficient, the past return is modeled to **increase** the prediction of the future return. The regression coefficients are estimated parameters, therefore **mvreg** also calculates an associated error term. This error term is called the Standard Error and is used to construct a confidence interval for what the true regression coefficient actually is.

The t-statistic is a ratio of the regression coefficient divided by its Standard Error. Similar to the F-ratio, the t-statistic has an associated p-value to help determine if it is significant or not. Later, I will use these measures to examine two of the **mvreg** output tables and reason through why one is a good predictor model and why the other is a poor predictor model (Table 9 and Table 10 in Section 4.3).

4 Optimal Time Period Analyses

4.1 3-Month Look Back Analysis

For a visualization of how the data is calculated, please refer back to Figure 1, substituting $x = 3$ for the independent variable calculations and $k = 3, 6, 12, 36$ for

Table 3: 3 Month look back Overall Model P-Values of F-Ratios for each Regression Model

Equation	XLV	XLI	XLV	XLP	XLB	XLK	XLU	XLE	XLF
$Return_{t+3}$	0.0233	0.1254	0.6354	0.0249	0.4942	0.0053	0.0034	0.4138	0.1679
$Return_{t+6}$	0.0303	0.4758	0.776	0.2424	0.9627	0.0000	0.042	0.6683	0.3185
$Return_{t+12}$	0.0191	0.3134	0.0027	0.0056	0.2342	0.0001	0.1194	0.0315	0.0449
$Return_{t+36}$	0.0197	0.1301	0.0024	0.1149	0.0203	0.0015	0.7081	0.5803	0.0063

the dependent variable calculations.

Table 3 shows the p-values of F-ratios for each sector and their prediction of each future return variable. As you will recall, smaller p-values indicate greater significance. This table shows that there are a handful of p-values less than 0.01 scattered across different sectors and time periods.

The highest overall model significance is found in the 6-month future return for the Technology sector, $Return_{t+6}(XLK)$. Table 4 shows XLK's regression coefficients and t-statistics for each independent variable, with the significant t-statistics in bold. From this table, we see that the intercept and the standard deviation contribute to this model's predictability. A significant intercept means that the average of the future returns is significantly different from zero. So, here the intercept indicates that $Return_{t+6}(XLK)$ tends to have positive returns on average for the time period given. The negative coefficient of $s_{t-3}(XLK)$ indicates that on average, high standard deviation has a significant negative impact on return over the next 6 months. Since the rest of the independent variables do not have significant t-statistics, we can conclude that their regression coefficients are not significantly different from 0.

Table 4: Regression Results for $Return_{t+6}(XLK)$ from 1/1/1999 - 11/30/2012

Sector	Intercept	$Return_{t-3}$	s_{t-3}	$Drawdown_{t-3}$	$LinRegSlope_{t-3}$	$EXRET_{t-3}$
XLK	0.09	-0.14	-1.71	-0.45	0.00	0.51
t-stat	3.53	-0.62	-5.07	-1.39	-0.09	2.17

The 3-month look back period did result in some predictive capability, but there is no single time period combination that stands out as a clear winner. 3 months is a short time period when it comes to past performance and is most likely the reason that the 3-month look back provides the smallest number of significant results compared to the longer time periods.

4.2 6-Month Look Back Analysis

The 6-month look back produced a higher number of significant models than the 3-month look back. Again, review Figure 1 for a visualization of the time periods used for data calculation, this time with $x = 6$.

From Table 5, we can quantify the overall model performance when compared to the other look back periods by comparing their respective p-values of the F-ratios. When comparing p-values by column we see sectors such as XLK and XLF, where the p-values are significantly small across all future time periods. This means XLK and XLF are good candidates for using this look back period. However, we are looking for a time period that gives small p-values across **all** of the sectors. Comparing the p-values across the rows, Table 5 shows that the 6-month look back is a much better predictor of $Return_{t+12}$ and $Return_{t+36}$ than it is a predictor of $Return_{t+3}$ and $Return_{t+6}$. The 12 and 36-month future returns are nearly all significant.

Table 5: 6-Month look back Overall Model P-Values for F-Ratios of Each Regression Model

Equation	XLV	XLI	XLY	XLP	XLB	XLK	XLU	XLE	XLF
$Return_{t+3}$	0.001	0.125	0.6069	0.2779	0.2055	0.0000	0.1104	0.2607	0.0002
$Return_{t+6}$	0.0977	0.0208	0.1064	0.1542	0.0129	0.0000	0.0059	0.0051	0.0000
$Return_{t+12}$	0.0858	0.0009	0.0002	0.0032	0.0004	0.0000	0.0009	0.0000	0.0000
$Return_{t+36}$	0.0012	0.0000	0.0000	0.0326	0.0001	0.0000	0.0007	0.0099	0.0008

Table 6: 6-Month look back and $Return_{t+12}$ - Regression Coefficients and T-Statistics						
Sector	Intercept	$Return_{t-6}$	s_{t-6}	$Drawdown_{t-6}$	$LinRegSlope_{t-6}$	$EXRET_{t-6}$
XLV	0.04	0.11	-0.64	-0.91	0.00	0.13
t-stat	1.28	0.41	-0.82	-2.55	1.00	1.04
XLI	-0.08	0.62	1.18	-1.14	0.00	-0.55
t-stat	-1.77	1.99	1.59	-3.54	1.03	-2.32
XLY	-0.07	0.03	1.77	-0.67	0.00	0.22
t-stat	-1.66	0.09	2.51	-2.10	1.49	1.19
XLP	0.01	0.40	0.30	-1.01	0.00	-0.28
t-stat	0.54	1.30	0.41	-2.94	0.41	-2.71
XLB	-0.01	0.20	0.45	-0.85	0.00	-0.24
t-stat	-0.22	0.77	0.67	-2.99	0.68	-1.42
XLK	0.25	-0.86	-3.83	0.21	0.00	0.76
t-stat	4.79	-2.65	-5.75	0.58	0.00	3.26
XLU	-0.01	1.03	0.08	-1.25	0.00	-0.59
t-stat	-0.23	3.17	0.09	-3.22	0.02	-3.92
XLE	0.27	0.71	-3.09	-0.35	0.00	-0.58
t-stat	4.52	2.71	-3.69	-1.09	-2.52	-2.92
XLF	-0.09	-1.02	1.51	-0.21	0.00	0.44
t-stat	-1.97	-3.21	2.17	-0.65	5.35	1.77

Table 6 shows each of the nine sectors' regression coefficients and t-statistics from the $Return_{t+12}$ model. If we count the number of significant t-statistics by column, we see that $Drawdown_{t-6}$ is a significant independent variable across five of the nine sectors; each of them have a negative regression coefficient. In Section 3.1 I defined that drawdown is either negative or 0, so these models are indicating that on average, drawdowns over the past 6 months have a significant positive impact on the 12-month future return.

$EXRET_{t-6}$ is also a significant independent variable across five of the nine sectors. However, depending on the sector, the regression coefficient is positive and negative. A positive regression coefficient implies that an asset that has outperformed the S&P 500 for the last 6 months is likely to continue to do so over the next 12-months. A negative regression coefficient implies that an asset that has

outperformed the S&P 500 for the last 6 months will tend to underperform in the next 12-months. This outcome gives an excellent reason why each sector has its own regression model; the independent variables do not affect each sector in the same manner.

When comparing the rows in Table 6, note that XLE and XLK have the highest number of significant regression coefficients. This doesn't come as too much of a surprise since these two sectors also had significant p-values in Table 5.

4.3 12-Month Look Back Analysis

So far, increasing the length of the look back period has increased the number of significant models. This will continue to be the case with the 12-month look back period.

The overall model p-values of the F-ratios for each of the dependent variables are displayed in Table 7 . By comparing the p-values across the columns, we see a high number of sectors that have significant p-values, regardless of the future time period. If we compare the p-values by row, we see that the two longer return predictions have the smallest p-values overall.

Table 8 gives an in-depth look at the independent variables used in the 12-month future return model. $Drawdown_{t-12}$ and $EXRET_{t-12}$ appear most frequently.

Table 7: 12-Month look back Overall Model P-Values for F-Ratios of Each Regression Model

Equation	XLV	XLI	XLY	XLP	XLB	XLK	XLU	XLE	XLF
$Return_{t+3}$	0.1128	0.0355	0.0923	0.0031	0.06	0.0063	0.0245	0.0000	0.0124
$Return_{t+6}$	0.4154	0.0014	0.0018	0.0001	0.0004	0.0001	0.0000	0.0000	0.0000
$Return_{t+12}$	0.0009	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
$Return_{t+36}$	0.0001	0.0000	0.0000	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000

Table 8: 12-Month look back and $Return_{t+12}$ - Regression Coefficients and T-Statistics

Sector	Intercept	$Return_{t-12}$	s_{t-12}	$Drawdown_{t-12}$	$LinRegSlope_{t-12}$	$EXRET_{t-12}$
XLV	0.10	-0.14	-1.94	-0.67	0.00	0.29
t-stat	2.17	-0.64	-1.96	-2.50	2.68	2.63
XLI	-0.17	0.91	2.23	-1.17	0.00	-0.93
t-stat	-3.36	4.05	2.61	-4.62	-0.48	-4.26
XLY	-0.17	0.00	3.14	-0.82	0.00	0.14
t-stat	-3.47	0.00	3.56	-3.55	2.59	1.02
XLP	-0.01	0.50	-0.26	-1.52	0.00	-0.27
t-stat	-0.38	2.24	-0.34	-6.17	0.95	-3.78
XLB	-0.05	0.38	1.48	-0.50	0.00	-0.34
t-stat	-0.81	1.90	1.74	-2.09	-1.42	-2.80
XLK	0.35	-0.78	-4.96	0.25	0.00	0.56
t-stat	5.48	-3.34	-5.00	0.85	-0.62	2.96
XLU	-0.07	1.26	0.65	-1.35	0.00	-0.89
t-stat	-1.37	5.26	0.64	-4.82	-1.51	-7.60
XLE	0.29	1.17	-3.74	-0.76	0.00	-1.04
t-stat	3.52	6.38	-3.05	-2.90	-5.65	-7.04
XLF	-0.12	-0.54	1.59	-0.41	0.00	0.37
t-stat	-2.43	-2.09	2.04	-1.49	3.89	2.05

These are the same two variables that had statistical significance in the 6-month look back period. $Drawdown_{t-12}$ is significant in seven of the nine sectors and $EXRET_{t-12}$ is significant in six. Similar to what we found in the 6-month look back analysis, the regression coefficients of $Drawdown_{t-12}$ are all negative and the regression coefficients for $EXRET_{t-12}$ are positive and negative.

Next, I will show the output computed by **mvreg** for XLE. The top section of Table 9 shows the overall model goodness of fit statistics (RMSE, R^2 , F-ratio and p-value) when using the past 12 months to predict the next 6 months. The p-value of the F-ratio is highly significant, which suggests that this model has predictive capability. The bottom section of Table 9 shows the independent variables and their goodness of fit statistics (Regression Coefficient, Standard Error, t-statistic and p-value, and a 95% confidence interval for the true regression coefficient). All of the

Table 9: **mvreg** - 12-Month look back and $Return_{t+6}(XLE)$

XLE	12-Month look back					
Overall Model	Obs.	Parms	RMSE	R-sq	F	P
$Return_{t+6}(XLE)$	156	6	0.18947	0.3851	18.78779	0.0000

$Return_{t+6}(XLE)$	Reg. Coef.	Std. Err.	t	$P > t $	[95% Conf.	Interval]
$Return_{t-12}$	1.17	0.184	6.38	0.0000	0.81	1.54
s_{t-12}	-3.74	1.228	-3.05	0.0030	-6.17	-1.32
$Drawdown_{t-12}$	-0.76	0.261	-2.90	0.0040	-1.27	-0.24
$LinRegSlope_{t-12}$	-0.0002774	0.00	-5.65	0.0000	-0.0003744	-0.0001805
$EXRET_{t-12}$	-1.04	0.147	-7.04	0.0000	-1.33	-0.75
Intercept	0.29	0.083	3.52	0.0010	0.13	0.46

p-values of these independent variables are significantly contributing to the model.

In contrast, I present the **mvreg** output for XLV in Table 10 . Again, the top section shows the overall model goodness of fit statistics and the bottom section shows the statistics of the independent variables. The p-value of the F-ratio of this model is **not** significant. This is enough to warrant exclusion from any predictive modeling, but if we look at the independent variables anyway, we see from the column labeled “ $P > |t|$ ” that each of the p-values are too large to be considered significant, as expected.

Table 10: **mvreg** - 12-Month look back and $Return_{t+6}(XLV)$

XLV	12-Month look back					
Overall Model	Obs.	Parms	RMSE	R-sq	F	P
$Return_{t+6}(XLV)$	156	6	0.09703	0.0325	1.00747	0.4154

$Return_{t+6}(XLV)$	Reg. Coef.	Std. Err.	t	$P > t $	[95% Conf.	Interval]
$Return_{t-12}$	-0.14	0.169	-0.82	0.4130	-0.47	0.19
s_{t-12}	-0.33	0.762	-0.43	0.6690	-1.83	1.18
$Drawdown_{t-12}$	-0.15	0.207	-0.72	0.4750	-0.56	0.26
$LinRegSlope_{t-12}$	0.0001237	0.00	1.15	0.2540	-0.0000896	0.0003371
$EXRET_{t-12}$	0.09	0.084	1.05	0.2940	-0.08	0.26
Intercept	0.03	0.034	0.88	0.3830	-0.04	0.10

Table 11: 36-Month look back R^2 Values for $Return_{t+12}$ and $Return_{t+36}$

Equation	XLV	XLI	XLY	XLP	XLB	XLK	XLU	XLE	XLF
$Return_{t+12}$	0.1284	0.2520	0.2987	0.2164	0.1892	0.1000	0.2039	0.3733	0.2786
$Return_{t+36}$	0.1605	0.4505	0.5402	0.5503	0.2118	0.1645	0.2711	0.2896	0.2337

4.4 36-Month Look Back Analysis

The 36-month look back gives results consistent with the 12-month look back, but it doesn't have as many significant models.

Table 11 presents the R^2 values that are achieved using the 36-month look back to predict $Return_{t+12}$ and $Return_{t+36}$. The R^2 values were not presented in the previous analyses, but XLP, XLY, and XLI achieve the three highest R^2 values in this paper, so we will take a look at them here. Using the past 36 months to calculate the independent variables resulted in models that were able to explain around 50% of the variance found in the 36 month future return of XLP, XLY, and XLI.

Table 12: 36-Month look back Overall Model P-Values for F-Ratios of each Regression Model

Equation	XLV	XLI	XLY	XLP	XLB	XLK	XLU	XLE	XLF
$Return_{t+3}$	0.0792	0.0645	0.0764	0.0524	0.0068	0.0135	0.2011	0.0002	0.0063
$Return_{t+6}$	0.7404	0.0155	0.0048	0.0096	0.0000	0.0818	0.0005	0.0000	0.0000
$Return_{t+12}$	0.0423	0.0000	0.0000	0.0000	0.0000	0.0196	0.0000	0.0000	0.0000
$Return_{t+36}$	0.0000	0.0000	0.0000	0.0000	0.0000	0.0004	0.0000	0.0000	0.0000

The p-values found in Table 12 explain the overall model significance. We see that the two longer future return models have more significance than the shorter two purely by counting the number of significant p-values.

Table 13 shows the regression coefficients and t-statistics from the 36-month look back period prediction for $Return_{t+12}$. As we saw in earlier analyses, $Drawdown_{t-x}$ and $EXRET_{t-x}$ play an important roll in the forward returns of most of the sectors. They are significant in five out of the nine sectors in the same manner that they were in both the 6-month and 12-month look back periods.

Table 13: 36-Month look back and $Return_{t+12}$ - Regression Coefficients and T-Statistics

Sector	Intercept	$Return_{t-36}$	s_{t-36}	$Drawdown_{t-36}$	$LinRegSlope_{t-36}$	$EXRET_{t-36}$
XLV	0.07	-0.07	-0.89	-0.60	0.00	0.22
t-stat	1.38	-0.3	-0.76	-2	1.96	1.7
XLI	-0.17	0.88	3.05	-0.95	0.00	-1.01
t-stat	-3.28	3.46	3.21	-3.33	-0.73	-3.47
XLY	-0.16	-0.05	3.21	-0.68	0.00	0.28
t-stat	-2.25	-0.21	2.26	-2.47	1.96	1.07
XLP	-0.04	0.56	1.00	-1.47	0.00	-0.20
t-stat	-1.03	2.27	0.79	-5.2	0.7	-1.71
XLB	-0.09	0.46	2.40	-0.45	0.00	-0.64
t-stat	-1.28	2.14	2.49	-1.76	-1.43	-3.6
XLK	0.06	0.02	-0.77	-0.59	0.00	0.31
t-stat	1.12	0.09	-0.86	-2.19	0.05	1.82
XLU	-0.11	1.18	1.82	-1.34	0.00	-0.57
t-stat	-1.72	4.6	1.35	-4.41	-0.86	-3.76
XLE	0.36	1.02	-4.01	-0.47	0.00	-0.95
t-stat	4.04	4.82	-3.1	-1.59	-5.71	-5.46
XLF	-0.09	-0.68	1.07	-0.77	0.00	0.98
t-stat	-1.66	-2.49	1.28	-2.57	4.38	4.14

4.5 Conclusion of Analyses

These analyses attached statistical significance to five independent variables, over varying lengths of time, to determine how history can be used to predict future performance. In total, there were 16 unique combinations of time periods analyzed for their predictive capabilities. I evaluated the significance of the F-ratio for each to determine overall model predictability. Then, I further evaluated the independent variables of models where the F-ratio was significant.

The count of overall model significance can be summarized as follows: The 3-month look back analysis produced a total of 9 significant F-ratios, the 6-month look back analysis produced 23 significant F-ratios, the 12-month look back analysis produced 29 significant F-ratios, and the 36-month look back analysis produced 25

significant F-ratios.

Since the 12-month look back generated the highest number of significant models, I will choose this time period as the optimal look back period. Within this look back period, there is high significance in both the 12-month future return and the 36-month future return. However, I have chosen the 12-month future return as the optimal future return period.

5 Trading Strategy

5.1 Select Sector SPDR ETF Sector Rotation - Outline

Here, I give an outline of the rules for selecting the sectors, discuss the allocations and how they can change from month to month, and highlight the performance of the strategy compared to the S&P 500. This strategy will use an equal-weighted allocation method. An analysis of different allocation methods is outside the scope of this paper.

The decision for the sector allocation will follow the same rules-based process at the beginning of each month. By following a set of rules, we are able to take emotion (or any other subjective factor) out of the decision making process. This gives the best chance of being able to replicate the performance characteristics from our backtested strategy going forward. The rules are as follows:

1. On the first day of the month, calculate the independent variables for each sector using the past 12 months.
2. Plug each of these independent variables, along with the regression coefficients,

into the regression formula.

3. If the result is positive, allocate $1/9^{th}$ of the portfolio to that sector.
4. The sector rotation strategy will hold the sectors with positive regression results (future return predictions) for the entire month.

For instance, if the model results in positive values for 6 sectors, the strategy will invest in those 6 each at 11.1% (approximately $1/9^{th}$) each for a portfolio that is 66.6% invested in equities and 33.4% invested in cash.

Table 14: Regression Coefficients for Calculating $Return_{t+12}$

Sector	β_0	β_1	β_2	β_3	β_4	β_5
Ind. Var.		$Return_{t-12}$	s_{t-12}	$Drawdown_{t-12}$	$LinRegSlope_{t-12}$	$EXRET_{t-12}$
XLV	0	0	0	-0.6702156	0.0003753	0.2874347
XLI	-0.1666002	0.9111505	2.227321	-1.165288	0	-0.9317845
XLY	-0.1738631	0	3.13873	-0.8249454	0.000382	0
XLP	-0.0114972	0	0	-1.519858	0	-0.2706815
XLB	-0.0494748	0.3768665	1.475752	-0.4973565	-0.0001328	-0.339808
XLK	0.3515096	-0.7801845	-4.960361	0	0	0.5636693
XLU	0	1.261372	0	-1.354802	0	-0.8863592
XLE	0.2928439	1.17492	-3.744959	-0.7553954	-0.0002774	-1.035919
XLF	-0.1212807	0	0	0	.0008231	0

Table 14 contains the significant regression coefficients determined by **mvreg**. For example, $Return_{t+12}(XLV)$ will be calculated each month using the equation

$$Return_{t+12}(XLV) = -0.67021(Drawdown_{t-12}) + 0.0003753(LinRegSlope_{t-12}) + 0.28743(EXRET_{t-12})$$

The resulting 12 month future return prediction for the Healthcare sector is a function of three of our five tested independent variables. Each sector will follow this format with its respective regression coefficients and independent variables.

5.2 Select Sector SPDR ETF Sector Rotation - Performance

The monthly return performance data presented in this section is based on the Select Sector SPDR ETF sector rotation strategy (“Strategy”) outlined in Section 5.1. The Strategy’s monthly returns were imported into Morningstar Direct in order to show performance compared to the S&P 500 Index. Performances are shown gross of fees. The in-sample date ends on 11/30/2012, performance shown after that date is out-of-sample⁹.

Table 15 gives the performance of the Strategy benchmarked to the S&P 500 for the full time period (1/1/2000 to 11/30/2015). On a risk adjusted return basis, as shown by the Sharpe ratio¹⁰, alpha¹¹, and beta¹², the Strategy outperformed the S&P 500 Index. It also achieved a lower annualized standard deviation and a higher annualized return. The max drawdown and worst month are both considerably better as well. The drawdown chart in Appendix A clearly shows a pattern of downside protection over the past 15 years.

The up capture ratio for the Strategy is 72.29%. This means that when the S&P

⁹Out-of-sample indicates a time period that was not used in determining the regression coefficients

¹⁰Sharpe-Ratio is calculated as the annualized return less the risk-free rate and then divided by the annualized standard deviation. In this paper are taking the risk-free rate to be 0.

¹¹An alpha value of 0 indicates an asset that is perfectly tracking its benchmark.

¹²Beta is an indication of how an asset’s price will move in response to the benchmark.

Table 15: Performance Statistics 1/1/2000 - 11/30/2015

	Return	Stdev	Sharpe Ratio	Alpha	Beta
Sector Rotation	7.04	10.06	0.55	3.43	0.60
S&P 500 TR USD	4.19	15.17	0.23	0.00	1.00

	Up Capture	Down Capture	Max Drawdown	Worst Month	Correlation
Sector Rotation	72.29	52.25	-28.90	-11.07	0.91
S&P 500 TR USD	100.00	100.00	-50.95	-16.79	1.00

500 has a positive monthly return, the Strategy, on average, is up about 72% as much as the S&P 500. The down capture ratio for the Sector Rotation is 52.25%. This means that when the S&P 500 has a negative monthly return, the Strategy, on average, is down only half of the S&P 500. These two measures provide evidence that the Strategy offers protection in periods of market decline, while still participating in periods of market growth.

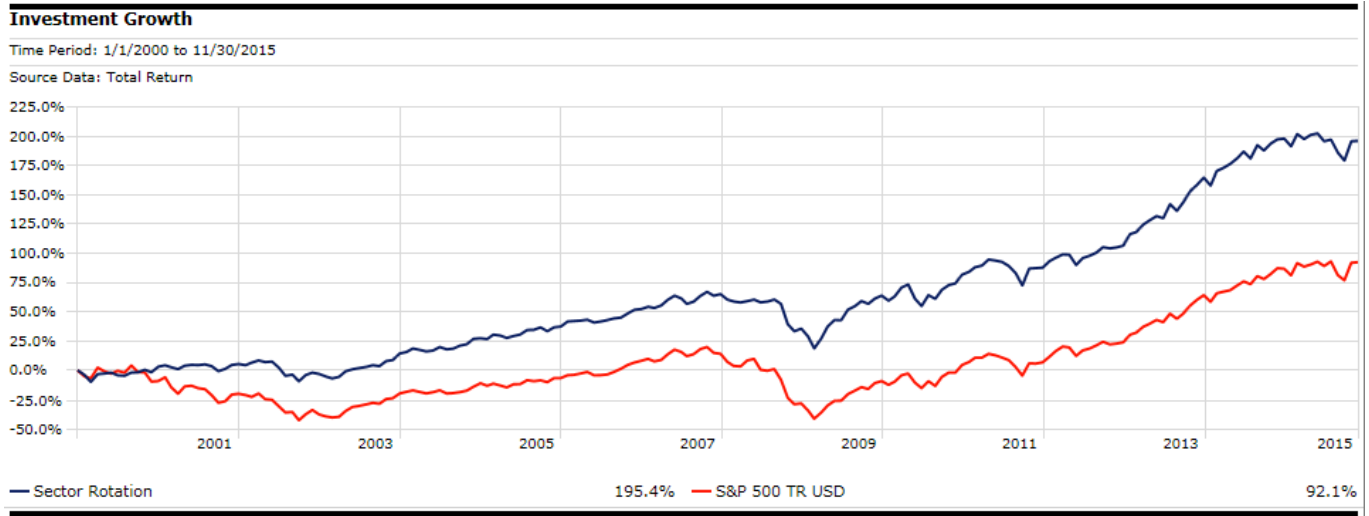
The alpha of the Strategy over this time period is positive. An alpha of 3.43% means that the Strategy is adding value to the S&P 500 by about 3.43% per year.

The beta value for the Strategy is 0.60. Beta less than 1 indicates a strategy that is less volatile than its benchmark and beta greater than 1 means the strategy is more volatile than its benchmark. For the Strategy, we can expect the price to move with about 40% ($1 - 0.60 = 0.40$) of the volatility of the S&P 500, on average.

Correlation can range between -1 and +1, with +1 implying that as an asset moves up and down, the other asset moves in lockstep. The Strategy's correlation to the S&P 500 is measured at 0.91. This high correlation is achieved while this portfolio is, on average, 68% invested in the Select Sector SPDR ETFs.

Figure 2 shows a chart of the investment growth, given as a percentage, generated by the Strategy and the S&P 500 Index. This time period covers a total of 191 months. The Strategy outperformed the S&P 500 in 96 of the 191 months (50.26%) by achieving a cumulative return of 195.4% vs the S&P 500's 92.1%. In the months where the Strategy outperforms, it does so on average by about 1.65% and when it underperforms it does so by about 1.33%. Next, let's take a look at the performance of the Strategy over the last three years to see if the model can be verified by out-

Figure 2: Percentage Growth Chart: Sector Rotation vs S&P 500 Index 1/1/2000 - 11/30/2015



of-sample data.

Table 16 gives the performance of the Strategy benchmarked to the S&P 500 beginning 12/1/2012 and ending 11/30/2015. Over the full time period, which included several market cycles, we saw that the strategy delivered superior performance to the S&P 500 in several risk adjusted metrics. For the last three years the market has been in a low volatility, upward trend with very few drawdowns. This is a tough market environment for a trend following approach. Yet, the Strategy still delivered positive alpha, reduced beta, and maintained the same Sharpe ratio as the S&P 500. This result adds significance to our regression models, bolstering the argument that this is not a statistical fluke and that multivariate regression is able to pick up on

Table 16: Performance Statistics 12/1/2012 - 11/30/2015

	Return	Stdev	Sharpe Ratio	Alpha	Beta
Sector Rotation	13.04	8.60	1.47	0.34	0.79
S&P 500 TR USD	16.09	10.49	1.48	0.00	1.00

	Up Capture	Down Capture	Max Drawdown	Worst Month	Correlation
Sector Rotation	81.69	81.67	-7.65	-3.71	0.97
S&P 500 TR USD	100.00	100.00	-8.36	-6.03	1.00

real trends in the data.

It is important that a backtested portfolio include results that are verified by out-of-sample data (also called “forward testing”). Forward testing helps to minimize “hand-picking” the best results from the past to create a strategy that backtests favorably.

To further analyze the risk adjusted returns of the Strategy compared to the S&P 500, the Sharpe ratio is presented in a 3-year window on a rolling 3-month shift in Appendix A. The Strategy has a higher 3-year Sharpe ratio in 47 of the 52 data points. Note, however, that Sharpe ratio as an evaluating measure comes into question when the calculated value is negative [Israelsen, 2009]. When I remove those that are negative for each of the portfolios the Strategy has a higher (positive) Sharpe ratio in 31 of the 36 data points .

6 Summary

With this paper, I have shown how to identify relationships among the Select Sector SPDR ETF past performance statistics and future returns using multivariate regression analysis. I interpreted this analysis in a way so as to make it accessible to a variety of active investment practitioners.

I began with a brief explanation of the assets to be used as representation for the sectors of the S&P 500 Index, namely, the logarithmic, monthly returns of the Select Sector SPDR ETFs. I followed this explanation with a description of the input variables to be used in the multivariate regression analysis. I chose the independent variables for their logical application to how investors evaluate assets. The dependent

variables were the future returns that occurred in the past.

Next, I established how these input variables interact when the length of the two time variables are adjusted. I found that this type of statistical analysis is sensitive to both the look back and future return time variables. The routine for evaluating each model's goodness of fit statistics involved first analyzing the overall model predictability, given by the F-ratio. Then, if this ratio was of significance, as measured by the p-value, I delved deeper into the model to take a look at the regression coefficients and t-statistics of the independent variables. This made it possible to identify trends in significant variables among the different sectors.

I employed the trends identified by the analysis in an actively managed US Large Cap Equity sector rotation. The performance that resulted from this trading system beat the S&P 500 Index according to several risk adjusted performance measures, calculated both in-sample and out-of-sample. Conclusively, I have shown that a rules-based trading system, supplemented with multivariate regression, is a viable alternative to investing in a passive index. This approach is not limited to these assets, input variables or time periods, and is merely a glimpse of the benefit that multivariate regression can provide to an active investment manager.

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Appendix A

Figure 3: 3-Year Window by 3-Month Shift: Sharpe Ratio 1/1/2000 - 11/30/2015

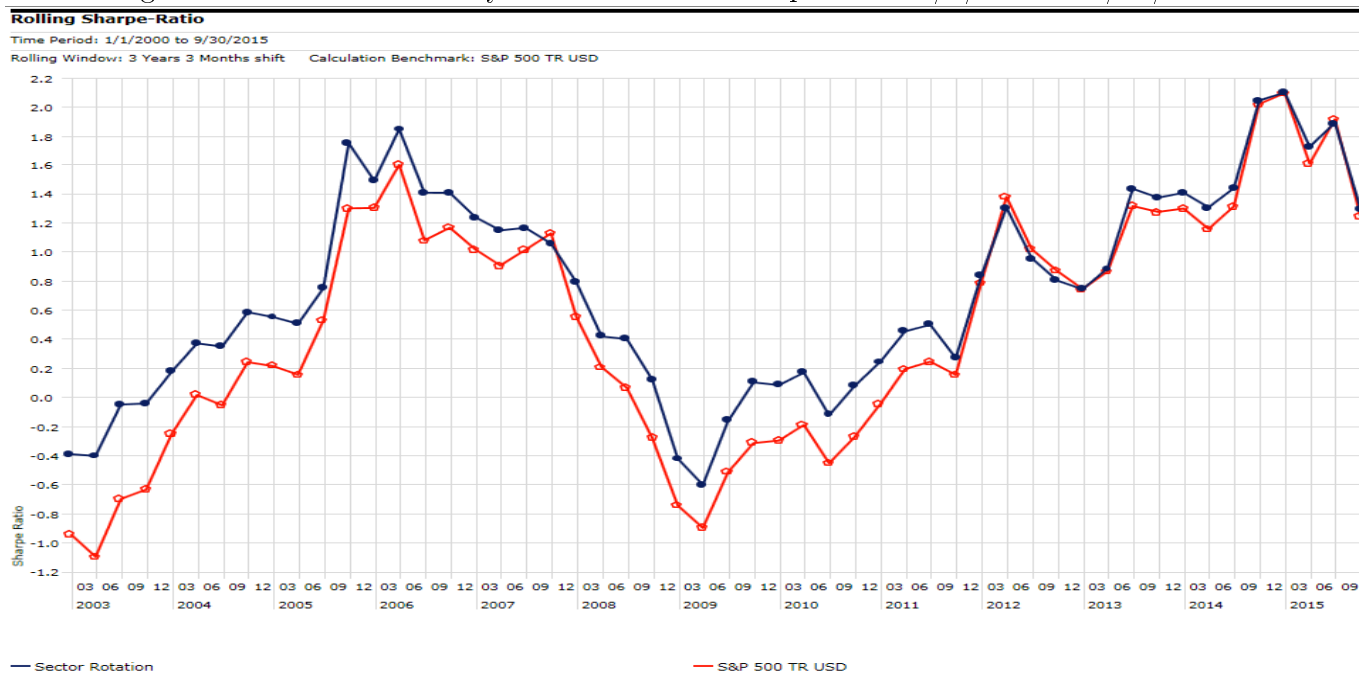


Figure 4: Sector Rotation Drawdown vs S&P 500 Drawdown

